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ABSTRACT

Numerous cosmic ray propagation and acceleration problems require knowledge of the propagation speed of relativistic particles through an ambient plasma. Previous calculations have indicated that self-generated turbulence scatters relativistic particles and reduces their bulk streaming velocity to the Alfven speed. This result has been incorporated into all currently prominent theories of cosmic ray acceleration and propagation. We, however, demonstrate that super-Alfvenic propagation is indeed possible for a wide range of physical parameters. This fact dramatically affects the predictions of these models.

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A thorough understanding of the physical processes which affect the streaming of relativistic particles through a magnetized thermal plasma is crucial to the problem of the origin of cosmic rays and has great importance for a number of astrophysical problems. In recent years, our observational knowledge of astrophysical situations where particle (cosmic ray) streaming is important has increased tremendously, thereby underscoring the importance of such an understanding. Specifically, these fundamental processes must be understood in order that one can construct self-consistent particle acceleration models involving the repeated scattering of relativistic particles between a shock front, or turbulence in the wake of the shock, and scattering centers in the pre-shock medium.[1,2,3] An even more basic related question is whether cosmic rays can escape acceleration regions (such as supernova remnants) without experiencing disastrous energy losses.[4]

A number of authors[5,6] have suggested that relativistic particles cannot stream freely along an ambient magnetic field, but will be limited to a mean parallel propagation velocity on the order of the Alfven speed. This occurs because the streaming particles amplify the pre-existing level of Alfven waves, which in turn resonantly scatter the particles and reduce their streaming velocity. In order to see how dramatically this idea has affected theoretical thought on the origin of cosmic rays, one need only look at the proceedings of the recent La Jolla Institute Conference on Particle Acceleration.[7]

The theoretical picture of trapping at the Alfven speed however, seems to contradict observations pertaining to a number of astrophysical plasmas (e.g., in supernova remnants, clusters of galaxies, extragalactic radio sources, etc.) which strongly indicate that particles do, in fact, propagate at speeds significantly greater than the Alfven speed. A resolution of this paradox for $\beta > 1$ plasmas has recently been suggested. [8]

However, in this note, we present a more fundamental solution to the problem (and we deal with $\beta \ll 1$ plasmas) which is applicable to plasmas with parameters which are expected to exist in regions of particle acceleration. Specifically, we find that in regions where cosmic ray pressure is significant,[10,11] the mean streaming velocity of the cosmic rays can be much greater than the local Alfven speed, V_A. This result stems from the fact that as the energy density of the relativistically streaming particles increases, so must the phase and group velocities of the destabilized waves. We also indicate that the cosmic ray particles could become trapped by these super-Alfvenic streaming modes, resulting in highly super-Alfvenic particle propagation. In fact, the resultant particle propagation speed can be high enough to significantly reduce the efficiency of particle acceleration mechanisms which rely upon the compression of the cosmic ray distribution resulting from multiple shock crossings.[1,2,3] In such models the repeated shock crossings are occasioned by upstream resonant particle scattering by waves which are traveling at a speed (i.e., the Alfven speed) which is much slower than the speed of the shock. Hence, in the shock frame, the shock and the upstream scattering centers are converging, and the cosmic rays are compressed. If, as we show, the particles "outrun" the shock, the particle energy gain is limited to that obtained in a single shock encounter. Furthermore, even if the upstream wave speed is only of the same order as the shock speed, then the energy gained per shock crossing is also reduced, since the rate of cosmic ray "compression" ($v_{shock} - v_{phase}$) is diminished.

Three components are assumed to comprise the system, viz., a cold background of electrons and ions, and relativistic ions (these are normally not included in the dielectric because of low number density) which are streaming parallel to an ambient field $\vec{B}$. We further assume that the system is infinite and homogeneous with zero net charge and zero net current. Our assumption of zero net current stems from the expectation that the cold background electrons will respond to an inductive "back" emf associated with the relativistic ions. This back emf will in turn drive a return current of electrons, with a negligibly small drift velocity, which results in approximate current neutralization. A simple model distribution function for the relativistic ions which is

separable in energy E and the cosine of the pitch angle (μ) will be incorporated into our analysis, and is given by

$$f(E, \mu) = \frac{\epsilon}{2\pi} F(E) g(\mu) \tag{1}$$

with $\epsilon = n_R/n_e$ and

$$F(E) = \begin{cases} 0 & E < E_* \\ (S-1)c^3 E_*^{S-1} E^{-(S+1)} & E \geq E_* \end{cases} \tag{2}$$

and

$$g(\mu) = 1/2 (1 + 3 <\mu> \mu) \tag{3}$$

where n_r is the number density of the relativistically streaming ions which stream at a velocity $v = <\mu> c$ and where the "cutoff" energy E_* is associated with a relativistic $\gamma_* = E_*/m_i c^2 \gg 1$. The plasma dielectric D for low-frequency electromagnetic waves propagating parallel to the ambient magnetic field is given by

$$D = k^2 c^2/\omega^2 - c^2/V_A^2 - D_R = 0 \tag{4}$$

with

$$D_R = \pi m_i c \left(\frac{\omega_{pi}^2}{\omega}\right) \int_0^\infty \left(\frac{E}{c}\right)^2 dE \int_{-1}^1 (1 - \mu^2) d\mu \left[\frac{\dfrac{\partial f}{\partial E} + \dfrac{1}{E}\left(\dfrac{kc}{\omega} - \mu\right) \dfrac{\partial f}{\partial \mu}}{\omega - kc\mu + \dfrac{P\Omega_i}{\gamma}} \right] \tag{5}$$

(cf. Montgomery and Tidman,[11] Equation 10.4, transformed to E, μ space) representing the contribution from the relativistically streaming ions. In equation (5) P = ± 1 represents the sense of polarization ((+) = right handed). The above dispersion relation (i.e., equation (4)) has been solved numerically assuming equations (1)-(3) and equation (4).[10] The relevant results are illustrated in Figure 1. The ratio of the unstable waves phase velocity to the Alfven speed is given by the solid curve and the ratio of the growth rate to the relativistic ions cutoff gyrofrequency ($\Omega_* = \Omega_i/\gamma_*$) is given by the dashed curve. Note that the phase velocity and growth rate both maximize at a wavenumber which is associated with the "super-Alfvenic streaming mode" (i.e., $kc/\Omega_* = 1$). This follows from the fact that the ratio of the relativistic particles' dielectric to the background plasma's dielectric becomes larger than unity (i.e., $V_A^2 D_R /c^2 \gg 1$) for

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wavenumbers of the order of Ω_*/c. Our results also clearly demonstrate that Alfven waves, which are found at both high and low wavenumbers, are relatively stable compared to the super-Alfvenic streaming mode. It is important to note that this results from the fact that the majority of the cosmic rays are resonant with the streaming mode, as is indicated by the shaded region in Figure 1. The shading represents the number of particles resonant with waves of a given wavenumber, with darker shading corresponding to a larger number. Since the resonance condition, $kc/\Omega_* = \gamma_*/\mu\gamma$, depends upon both the pitch angle and particle energy and, since the number of particles falls off with increasing γ and decreasing μ, it's clear that the bulk of the cosmic rays are resonant with the waves with the highest phase velocity (i.e., the streaming mode).

Our numerical results show that accurate analytic approximations to the solution can also be obtained through use of Plemelj's formula (even though $\Gamma > \omega$), since $\Gamma \ll \Omega_*$. We find the following normalized form for the dispersion relation:

$$\bar{\omega}^2 - \bar{\Gamma}^2 + A\bar{\omega} - B\bar{\Gamma} + C + i[2\bar{\omega}\bar{\Gamma} + B\bar{\omega} + A\bar{\Gamma} + D] = 0 \qquad (6)$$

where

$$\bar{\omega} = \frac{\omega}{\Omega_*}, \quad \bar{\Gamma} = \frac{\Gamma}{\Omega_*} \quad \text{and} \quad \Omega_* = \frac{\Omega_1}{\gamma_*}$$

and where

$$A = -\epsilon <\mu> \left(\frac{kc}{\Omega_*}\right) + P\gamma_* \epsilon \left[1 - \frac{(S-1)\,PN_1\,\Omega_*}{4kc}\right] \qquad (7)$$

$$B = \tau \left(\frac{S-1}{4}\right) \gamma_* \epsilon\, R_1 \left(\frac{\Omega_*}{kc}\right) \qquad (8)$$

$$C = -\left(\frac{kV_A}{\Omega_*}\right)^2 - \left[\frac{1}{\gamma_*}\frac{m_e}{m_1}\left(\frac{kV_A}{\Omega_*}\right)^2 - 3\left(\frac{S-1}{4}\right)<\mu>N_2\right]P\,\epsilon<\mu>\left(\frac{kc}{\Omega_*}\right)$$

$$\qquad (9)$$

$$+ \left[3\left(\frac{S-1}{4}\right)N_2 - P\frac{kc}{\Omega_*}\right]\gamma_*\epsilon<\mu>$$

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$$D = - 3\pi \left(\frac{S-1}{4}\right) \gamma_* \epsilon <\mu> R_2 \tag{10}$$

with:

$$N_\alpha = \int_0^1 w^{S-1} dw \int_{-1}^1 d\mu \, (1 - \mu^2) \left[\frac{l_\alpha(\mu)}{\dfrac{\omega}{kc} - \mu + \left(\dfrac{P\Omega_*}{kc}\right) w}\right] \tag{11}$$

$$R_\alpha = \int_0^{w_u} w^{S-1} dw \int_{-1}^1 d\mu \, (1 - \mu^2) \, l_\alpha(\mu) \, \delta\left(\frac{\omega}{kc} - \mu + \frac{P\Omega_*}{kc} w\right) \tag{12}$$

$$l_\alpha = \begin{cases} (S+2) + 3(S+3) <\mu> \mu & \alpha = 1 \\ 1 & \alpha = 2 \end{cases} \tag{13}$$

and

$$w_u = \begin{cases} \dfrac{kc}{\Omega_*} & \dfrac{kc}{\Omega_*} \leq 1 \\ 1 & \dfrac{kc}{\Omega_*} > 1 \end{cases} \tag{14}$$

From equation (6) we find that

$$\bar{\Gamma} = \frac{R\bar{\omega} + D}{-2\bar{\omega} + A} \tag{15}$$

and

$$\bar{\omega}^2 - \bar{\Gamma}^2 + A\bar{\omega} - B\bar{\Gamma} + C = 0 \tag{16}$$

Solving equations (15) and (16) for the wave of maximum growth (i.e., when $\dfrac{kc}{\Omega_*} \simeq 1$) we find that

$$\omega^{max} \approx kV_A \left(1 + \frac{\pi}{5} \gamma_* \epsilon <\mu> c^2 \left(\frac{kc}{\Omega_*}\right)^{\prime\prime} \left(\frac{1}{V_A^2}\right)\right)^{\prime\prime} \tag{17}$$

Curves of Γ_{max} versus ϵ for various $<\mu>$ are shown in Figure 2. Clearly, for $\gamma_* \epsilon <\mu> c^2 > V_A^2$ the phase velocity of this growing wave exceeds the Alfven speed. We note here,

however, that the region of applicability is limited to $\epsilon < V_A/c$; in order to extend the applicability of these calculations to $\epsilon > V_A/c$, a more realistic and detailed model of the electron return current would be needed.

We see then, that an initially relativistically streaming ($<\mu> \sim 1$) beam of cosmic rays (with large parallel momentum flux) produces waves with phase velocities, V_{ph}, much greater than V_A, to wit:

$$V_{ph} = \left(\frac{\pi}{5} \gamma_* \epsilon <\mu> \sqrt{kc/\Omega_*} \right)^{\frac{1}{2}} c \tag{18}$$

For the waves of maximum growth, $kc/\Omega_* \sim 1$. Therefore, to order of magnitude, the phase velocity of the super-Alfvenic streaming mode is given by

$$V_{ph} \sim c(\gamma_* \epsilon)^{\frac{1}{2}} \tag{19}$$

which for many cases of interest is significantly larger than the Alfven speed. In light of this fact, it is important to discuss how the distribution of streaming cosmic rays will be affected by the waves with V_{ph} given by equation (19). Note that quasi-linear relaxation is probably not the relevant relaxation process since the waves are coherent, i.e., the random phase approximation ($\Gamma^{-1} > t_{coherence} \sim 1/\Delta\omega$) does not hold. This is due to the fact that $\Gamma > \omega$ (c.f., Figure 2) and, with $\omega = kc\mu + \Omega_*$, we have $\Delta\omega = kc\Delta\mu + \mu c\Delta\omega/V_{ph}$ and therefore $\Delta\omega \sim \omega\Delta\mu/\mu \sim \omega$. Hence $\Gamma > \Delta\omega$, in contradiction to the random phase approximation.

The fact that the rapidly growing waves are highly coherent leads one to expect that electromagnetic wave trapping will occur, in analogy to the work of Davidson et al.[14] These authors find that trapping occurs when the linear growth time becomes comparable to the magnetic bounce time $t_B \sim 2\pi (\gamma_* m_\parallel c/ev_\parallel k\delta B)^{\frac{1}{2}}$, where δB is the wave magnetic field strength. We find that trapping of the cosmic rays by the super-Alfvenic streaming modes occurs when the wave level is $\delta B/B \sim (\Gamma/\Omega_*)^2/\mu \ll 1$. Therefore, a high wave level is not required in order that the cosmic rays become trapped in the magnetic potential well of the streaming mode. We therefore

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anticipate that the further development of this instability will proceed analogously to the well known non-linear development of an electrostatic beam-plasma instability (Drummond et al.[15]); the trapped cosmic rays will be carried along by the streaming mode which, as we have already demonstrated, propagates at super-Alfvenic speeds. We note here that it is not within the scope of this paper to study the details of such non-linear interactions. It is apparent, however, that non-linear effects (i.e., trapping) will dominate quasi-linear relaxation; these non-linear effects allow super-Alfvenic propagation of cosmic rays.

A specific example which illustrates the importance of these results is that of cosmic rays escaping from a young supernova remnant (e.g., Casseopia A exhibits strong observational evidence of ongoing particle acceleration). In this case $\epsilon \sim 10^{-6}$, $V_A/c \sim 7.27 \times 10^{-6}$ (for the ambient medium) and $\gamma_e \sim 10$ which, from equation (19), results in a super-Alfvenic streaming mode whose phase velocity $V_{ph} \sim 435\, V_A$. This velocity is comparable to that of the supernova shock, V_s, and hence the effect discussed here is expected to significantly impact theories of cosmic ray escape from the supernova and theories of particle acceleration which rely on acceleration at the shock front. Such theories[1,2,3] (e.g., Axford et al., Bell) assume that $V_{ph} \ll V_s$. However, since at least initially $V_{ph} \sim V_s$, the spectrum of cosmic rays, dN/dE, produced by these acceleration mechanisms must be steeper than the currently predicted case $\dfrac{d\log N}{d\log E} = -2$. Clearly, shock wave acceleration theories must be modified in order to discern how large this effect is.

In conclusion, we have applied the plasma physics of streaming modes to cosmic ray propagation and have demonstrated that the phase velocity of these waves is super-Alfvenic in a variety of cosmic settings, especially in sites of particle acceleration.[3] We have also indicated that the cosmic ray particles could very easily become trapped in the magnetic potential well of these streaming modes. The natural consequence of this trapping is that the cosmic ray particles are carried along by the streaming mode at highly super-Alfvenic speeds. The enhanced phase velocity of the streaming mode, as discussed here, as well as associated non-linear effects (e.g., trapping)
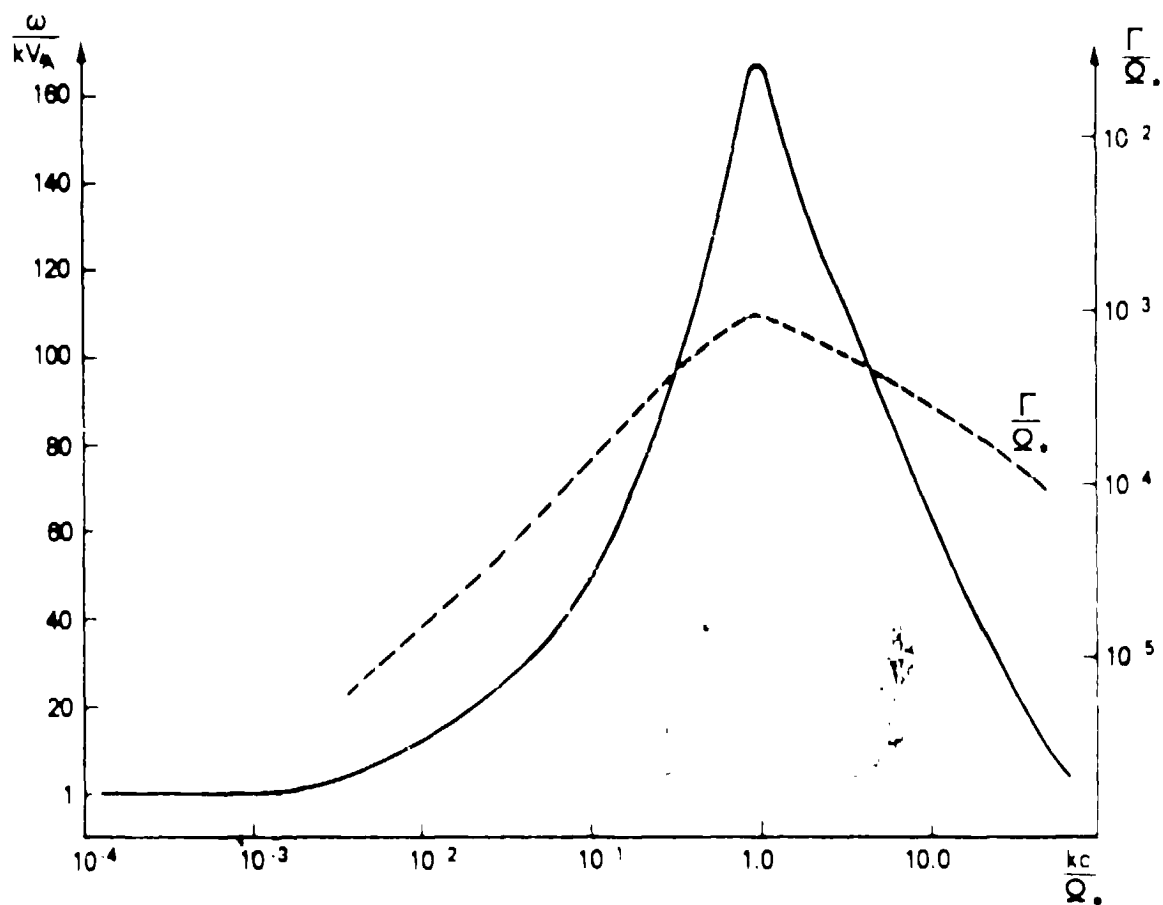
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Figure 1

Plot of the normalized phase velocity (solid curve) and growth rate (dashed curve) versus the wavenumber. The shaded region represents the number of particles that are resonant with waves of a given wavenumber. Typical supernova parameters have been used, i.e., $\epsilon \sim 10^{-6}$, $V_A/c \sim 7.27 \times 10^{-6}$ and $\gamma_o \sim 10$.
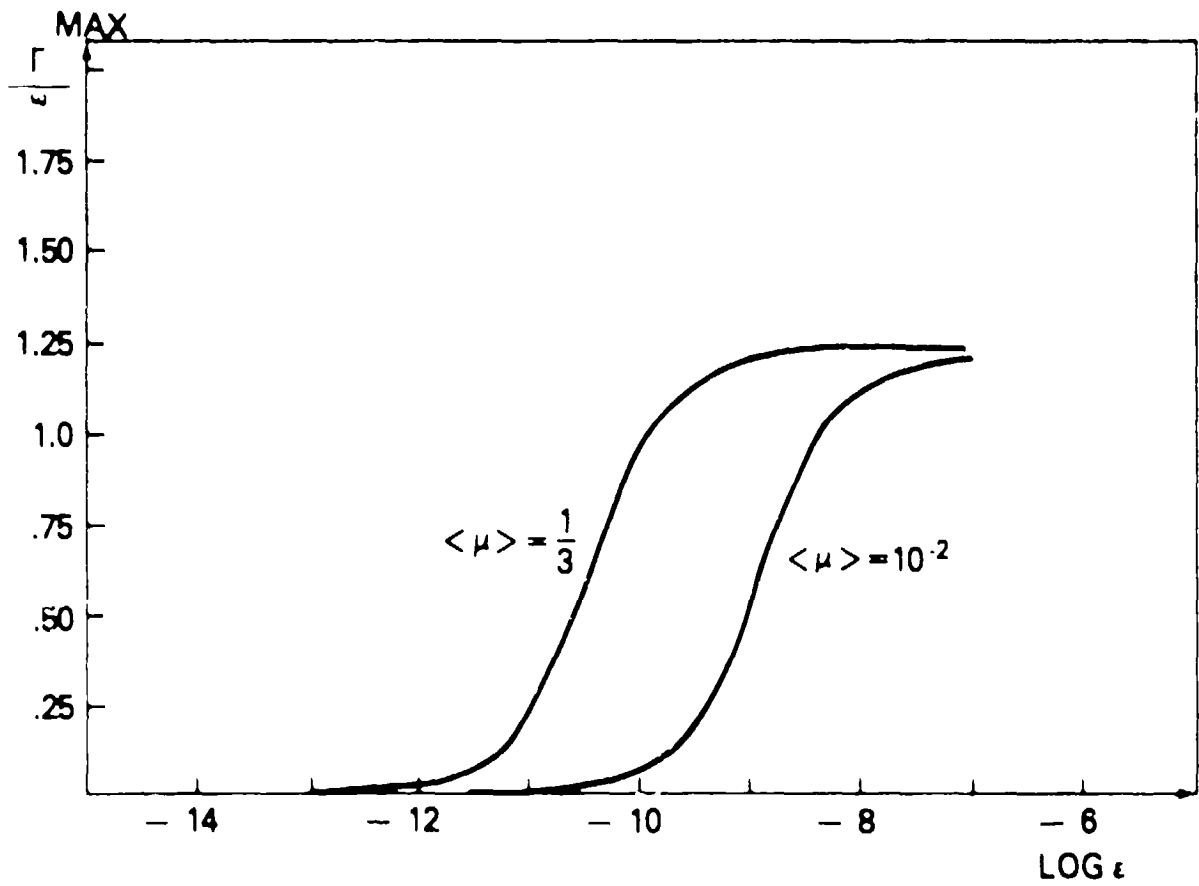
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Figure 2

Ratio of the maximum growth rate to the frequency, Γ^{MAX}/ω, versus the logarithm of the ratio of relativistic particle density to background electron density, LOG ϵ, for characteristic values of the ratio of the average streaming speed to the speed of light, $<\mu>$.

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will significantly alter current models of cosmic ray acceleration and propagation, clearly motivating more detailed investigations of the impact that our results will have on these models

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